THE HIGH SCHOOL FINALS

The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,
GET SET,...

2:30

INTEGRAL #1

$$\int \sin x \sqrt{\cos x}\, dx$$

2:30

INTEGRAL #1

$$\int \sin x \sqrt{\cos x}\, dx$$

$$= -\int \sqrt{u}\, du \quad \left[u = \cos x, \quad du = -\sin x\, dx\right]$$

$$= -\frac{2u^{3/2}}{3} + C$$

$$= \boxed{-\frac{2\cos^{3/2} x}{3} + C}$$

INTEGRAL #2

READY,
GET SET,...

2:30

INTEGRAL #2

$$\int \frac{1}{\sqrt{x}\,(2012+\sqrt{x})^5}\,dx$$

2:30

INTEGRAL #2

$$\int \frac{1}{\sqrt{x}\,(2012+\sqrt{x})^5}\,dx$$

$$= 2\int \frac{1}{u^5}\,du \qquad \left[u = 2012+\sqrt{x}, \quad du = \frac{1}{2\sqrt{x}}\,dx\right]$$

$$= 2\left(-\frac{1}{4u^4}\right) + C$$

$$= \boxed{-\frac{1}{2\,(2012+\sqrt{x})^4} + C}$$

INTEGRAL #3

READY,
GET SET,...

2:30

INTEGRAL #3

$$\int x\left(2x^3+1\right)^2\,dx$$

2:30

INTEGRAL #3

$$\int x\left(2x^3+1\right)^2\,dx$$

$$= \int x\left(4x^6+4x^3+1\right)\,dx$$

$$= \int \left(4x^7+4x^4+x\right)\,dx$$

$$= \boxed{\frac{x^8}{2}+\frac{4x^5}{5}+\frac{x^2}{2}+C}$$

INTEGRAL #4

READY,
GET SET,...

2:30

INTEGRAL #4

$$\int \frac{x+1}{(x^2+2x+2012)^9}$$

2:30

INTEGRAL #4

$$\int \frac{x+1}{(x^2+2x+2012)^9}$$

$$= \frac{1}{2}\int \frac{du}{u^9} \quad \left[u = x^2+2x+2012, \quad dx = 2(x+1)\,dx\right]$$

$$= \frac{1}{2}\left(-\frac{1}{8u^8}\right) + C$$

$$= \boxed{-\frac{1}{16(x^2+2x+2012)^8} + C}$$

INTEGRAL #5

READY,
GET SET,...

2:30

INTEGRAL #5

$$\int \frac{x}{\sqrt{7x^2+7}}\,dx$$

2:30

INTEGRAL #5

$$\int \frac{x}{\sqrt{7x^2+7}}\,dx$$

$$= \frac{1}{14}\int \frac{du}{\sqrt{u}} \quad \left[u = 7x^2+7, \quad du = 14x\,dx\right]$$

$$= \frac{1}{14}\cdot 2\sqrt{u} + C$$

$$= \boxed{\frac{\sqrt{7x^2+7}}{7} + C}$$

INTEGRAL #6

READY,
GET SET,...

2:30

INTEGRAL #6

$$\int \frac{1}{x^3}\sqrt[3]{1+\frac{1}{x^2}}\,dx$$

2:30

INTEGRAL #6

$$\int \frac{1}{x^3}\sqrt[3]{1+\frac{1}{x^2}}\,dx$$

$$= -\frac{1}{2}\int u^{1/3}\,du \qquad \left[u = 1+\frac{1}{x^2}, \quad du = -\frac{2}{x^3}\,dx\right]$$

$$= -\frac{1}{2}\cdot\frac{3u^{4/3}}{4} + C$$

$$= \boxed{-\frac{3}{8}\left(1+\frac{1}{x^2}\right)^{4/3} + C}$$

INTEGRAL #7

READY,
GET SET,...

2:30

INTEGRAL #7

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

2:30

INTEGRAL #7

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

$$= 2\int \sin u\,du \qquad \left[u = \sqrt{x}, \quad du = \frac{1}{2\sqrt{x}}\,dx\right]$$

$$= -2\cos u + C$$

$$= \boxed{-2\cos\sqrt{x} + C}$$

INTEGRAL #8

READY,
GET SET,...

2:30

INTEGRAL #8

$$\int \frac{\sin x - \cos x}{\sqrt{\sin x + \cos x}}\, dx$$

2:30

INTEGRAL #8

$$\int \frac{\sin x - \cos x}{\sqrt{\sin x + \cos x}}\, dx$$

$$= -\int \frac{du}{\sqrt{u}} \qquad \left[u = \sin x + \cos x, \quad du = (\cos x - \sin x)\, dx\right]$$

$$= -2\sqrt{u} + C$$

$$= \boxed{-2\sqrt{\sin x + \cos x} + C}$$

INTEGRAL #9

READY,
GET SET,...

2:30

INTEGRAL #9

$$\int x^5 \sqrt{x^3 + 1}\, dx$$

2:30

INTEGRAL #9

$$\int x^5\sqrt{x^3+1}\,dx$$

$$\left[\text{one of many possible subs: } u = x^3+1, \quad du = 3x^2\,dx\right]$$

$$= \frac{1}{3}\int (u-1)\sqrt{u}\,du = \frac{1}{3}\int \left(u^{3/2} - u^{1/2}\right)\,du$$

$$= \boxed{\frac{1}{3}\left(\frac{2(x^3+1)^{5/2}}{5} - \frac{2(x^3+1)^{3/2}}{3}\right) + C}$$

INTEGRAL #10

READY,
GET SET,...

2:30

INTEGRAL #10

$$\int (\sin x + \cos x)^2 \, dx$$

2:30

INTEGRAL #10

$$\int (\sin x + \cos x)^2 \, dx$$

$$= \int \left(\sin^2 x + 2 \sin x \cos x + \cos^2 x\right) \, dx$$

$$= \int (1 + 2 \sin x \cos x) \, dx \quad \left[= \int (1 + \sin 2x) \, dx\right]$$

$$= \boxed{x + \sin^2 x + C \quad \text{or} \quad x - \cos^2 x + C \quad \text{or} \quad x - \frac{\cos 2x}{2} + C}$$

INTEGRAL #11

READY,
GET SET,...

2:30

INTEGRAL #11

$$\int \frac{1}{x\sqrt{x}}\left(2+\frac{1}{\sqrt{x}}\right)^4 \, dx$$

2:30

INTEGRAL #11

$$\int \frac{1}{x\sqrt{x}}\left(2+\frac{1}{\sqrt{x}}\right)^4 dx$$

$$= -2\int u^4\, du \quad \left[u = 2+\frac{1}{\sqrt{x}}, \quad du = -\frac{1}{x\sqrt{x}}\right]$$

$$= \frac{-2u^5}{5} + C$$

$$= \boxed{-\frac{2}{5}\left(2+\frac{1}{\sqrt{x}}\right)^5 + C}$$

INTEGRAL #12

READY,
GET SET,...

2:30

INTEGRAL #12

$$\int \frac{\sin 2x}{\cos^3 x}\,dx$$

2:30

INTEGRAL #12

$$\int \frac{\sin 2x}{\cos^3 x}\,dx$$

$$= \int \frac{2\sin x \cos x}{\cos^3 x}\,dx = 2\int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx$$

$$= 2\int \sec x \tan x\,dx$$

$$= \boxed{2\sec x + C}$$