THE HIGH SCHOOL FINALS

The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,
GET SET,...

2:30

INTEGRAL #1

$$\int (x-2)(x+2)(x^2+4)\,dx$$

2:30

INTEGRAL #1

$$\int (x-2)(x+2)(x^2+4)\,dx$$

$$= \int (x^2-4)(x^2+4)\,dx$$

$$= \int (x^4-16)\,dx$$

$$= \boxed{\frac{x^5}{5} - 16x + C}$$

INTEGRAL #2

READY,
GET SET,...

2:30

INTEGRAL #2

$$\int x^2 \sin\left(x^3\right)\ dx$$

2:30

INTEGRAL #2

$$\int x^2 \sin\left(x^3\right)\, dx$$

$$= \frac{1}{3}\int \sin u \, du \quad \{u = x^3, \quad du = 3x^2\, dx\}$$

$$= \frac{1}{3}(-\cos u) + C$$

$$= \boxed{-\frac{\cos\left(x^3\right)}{3} + C}$$

INTEGRAL #3

READY,
GET SET,...

2:30

INTEGRAL #3

$$\int \left(\frac{x^2}{2} - \frac{2}{x^2} \right)^2 dx$$

2:30

INTEGRAL #3

$$\int \left(\frac{x^2}{2} - \frac{2}{x^2}\right)^2 dx$$

$$= \int \left(\frac{x^4}{4} - 2 + \frac{4}{x^4}\right) dx = \int \left(\frac{x^4}{4} - 2 + 4x^{-4}\right) dx$$

$$= \frac{x^5}{20} - 2x + \frac{4x^{-3}}{-3} + C$$

$$= \boxed{\frac{x^5}{20} - 2x - \frac{4}{3x^3} + C}$$

INTEGRAL #4

READY,
GET SET,...

2:30

INTEGRAL #4

$$\int \frac{\sin x}{\sqrt{2+\cos x}}\,dx$$

2:30

INTEGRAL #4

$$\int \frac{\sin x}{\sqrt{2+\cos x}}\,dx$$

$$= -\int \frac{1}{\sqrt{u}}\,du \quad \{u = 2+\cos x, \quad du = -\sin x\,dx\}$$

$$= -2\sqrt{u} + C$$

$$= \boxed{-2\sqrt{2+\cos x} + C}$$

INTEGRAL #5

READY,
GET SET,...

2:30

INTEGRAL #5

$$\int \frac{x}{e^x}\,dx$$

2:30

INTEGRAL #5

$$\int \frac{x}{e^x}\,dx$$

$$\left[\text{integrate by parts: } \begin{array}{rcl} u &=& x \\ du &=& dx \end{array}, \quad \begin{array}{rcl} dv &=& e^{-x}\,dx \\ v &=& -e^{-x} \end{array}\right]$$

$$= -xe^{-x} + \int e^{-x}\,dx$$

$$= \boxed{-xe^{-x} - e^{-x} + C}$$

INTEGRAL #6

READY,
GET SET,...

2:30

INTEGRAL #6

$$\int \frac{x+1}{(x^2+2x+2013)^5}\,dx$$

2:30

INTEGRAL #6

$$\int \frac{x+1}{\left(x^2+2x+2013\right)^5}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u^5}\,du$$

$$\left[u = x^2+2x+2013, \quad du = (2x+2)\,dx = 2(x+1)\,dx\right]$$

$$= -\frac{1}{8u^4} + C = \boxed{-\frac{1}{8\left(x^2+2x+2013\right)^4} + C}$$

INTEGRAL #7

READY,
GET SET,...

2:30

INTEGRAL #7

$$\int \frac{1}{\sqrt{x}\,(2013+\sqrt{x})^5}\,dx$$

2:30

INTEGRAL #7

$$\int \frac{1}{\sqrt{x}\,(2013+\sqrt{x})^5}\,dx$$

$$= 2\int \frac{1}{u^5}\,du \quad \left\{u = 2013+\sqrt{x}, \quad du = \frac{1}{2\sqrt{x}}\,dx\right\}$$

$$= 2\int u^{-5}\,du = \frac{2u^{-4}}{-4} + C = -\frac{1}{2u^4} + C$$

$$= \boxed{-\frac{1}{2\,(2013+\sqrt{x})^4} + C}$$

INTEGRAL #8

READY,
GET SET,...

2:30

INTEGRAL #8

$$\int \frac{\sec\sqrt{x}\tan\sqrt{x}}{\sqrt{x}}\,dx$$

2:30

INTEGRAL #8

$$\int \frac{\sec\sqrt{x}\tan\sqrt{x}}{\sqrt{x}}\,dx$$

$$= 2\int \sec u \tan u \, du \qquad \left[u = \sqrt{x}, \quad du = \frac{1}{2\sqrt{x}}\,dx\right]$$

$$= 2\sec u + C$$

$$= \boxed{2\sec\sqrt{x} + C}$$

INTEGRAL #9

READY,
GET SET,...

2:30

INTEGRAL #9

$$\int e^{2x}\sqrt{e^{2x}+1}\,dx$$

2:30

INTEGRAL #9

$$\int e^{2x}\sqrt{e^{2x}+1}\,dx$$

$$= \frac{1}{2}\int \sqrt{u}\,du \quad \left[\, u = e^{2x}+1, \quad du = 2e^{2x}\,dx \,\right]$$

$$= \frac{u^{3/2}}{3} + C$$

$$= \boxed{\frac{(e^{2x}+1)^{3/2}}{3} + C}$$

INTEGRAL #10

READY,
GET SET,...

2:30

INTEGRAL #10

$$\int (\sin x + \cos x)^2 \, dx$$

2:30

INTEGRAL #10

$$\int (\sin x + \cos x)^2 \, dx$$

$$= \int \left(\sin^2 x + 2 \sin x \cos x + \cos^2 x\right) \, dx$$

$$= \int (1 + 2 \sin x \cos x) \, dx = \int (1 + \sin 2x) \, dx$$

$$= \boxed{x - \frac{\cos 2x}{2} + C \quad \text{or} \quad x + \sin^2 x + C \quad \text{or} \quad x - \cos^2 x + C}$$

INTEGRAL #11

READY,
GET SET,...

2:30

INTEGRAL #11

$$\int \frac{1}{(20x+13)\sqrt{20x+13}}\,dx$$

2:30

INTEGRAL #11

$$\int \frac{1}{(20x+13)\sqrt{20x+13}}\,dx$$

$$= \int \frac{1}{(20x+13)^{3/2}}\,dx$$

$$= \frac{1}{20}\int u^{-3/2}\,du \quad [\,u = 20x+13, \quad du = 20\,dx\,]$$

$$= \frac{1}{20}\cdot\frac{u^{-1/2}}{-1/2} + C = \boxed{-\frac{1}{10\sqrt{20x+13}} + C}$$

INTEGRAL #12

READY,
GET SET,...

2:30

INTEGRAL #12

$$\int (\sin x - \cos x)(\sin x + \cos x)^2 \, dx$$

2:30

INTEGRAL #12

$$\int (\sin x - \cos x)(\sin x + \cos x)^2 \, dx$$

$$= -\int u^2 \, du \quad \left[u = \sin x + \cos x, \quad du = (\cos x - \sin x)\, dx\right]$$

$$= -\frac{u^3}{3} + C$$

$$= \boxed{-\frac{(\sin x + \cos x)^3}{3} + C}$$

INTEGRAL #13

READY,
GET SET,...

2:30

INTEGRAL #13

$$\int \cos x \cos 2x \, dx$$

2:30

INTEGRAL #13

$$\int \cos x \cos 2x \, dx$$

$$= \int \cos x \left(1 - 2\sin^2 x\right) dx$$

$$= -\int \left(\cos x - 2\sin^2 x \cos x\right) dx$$

$$= \boxed{-\sin x + \frac{2\sin^3 x}{3} + C}$$

INTEGRAL #14

READY,
GET SET,...

2:30

INTEGRAL #14

$$\int \frac{1}{x^3}\sqrt[3]{1+\frac{1}{x^2}}\,dx$$

2:30

INTEGRAL #14

$$\int \frac{1}{x^3}\sqrt[3]{1+\frac{1}{x^2}}\,dx$$

$$= -\frac{1}{2}\int u^{1/3}\,du \quad \left\{u = 1+\frac{1}{x^2}, \quad du = -\frac{2}{x^3}\,dx\right\}$$

$$= -\frac{1}{2}\cdot\frac{u^{4/3}}{4/3} + C$$

$$= \boxed{-\frac{3}{8}\left(1+\frac{1}{x^2}\right)^{4/3} + C}$$

INTEGRAL #15

READY,
GET SET,...

2:30

INTEGRAL #15

$$\int x \cos^2\left(x^2\right) \sin\left(x^2\right)\ dx$$

2:30

INTEGRAL #15

$$\int x \cos^2\left(x^2\right) \sin\left(x^2\right)\ dx$$

$$= -\frac{1}{2}\int u^2\, du \quad \left\{u = \cos\left(x^2\right), \quad du = -2x \sin\left(x^2\right)\ dx\right\}$$

$$= -\frac{u^3}{6} + C$$

$$= \boxed{-\frac{\cos^3\left(x^2\right)}{6} + C}$$