COLLEGE ROUND ONE

You will have **two minutes** to evaluate each of the fifteen definite integrals that will displayed one at a time on this screen. **All answers must be simplified.** At the end of the two minutes, all hands must go up and judges will grade your answers immediately. For each correct answer, you will receive one raffle ticket to be entered for prizes that will be drawn after dinner.

At most five participants will move to the finals – to be determined by the total number of correct answers and tiebreaking criteria if necessary. **Everyone moving to the finals will receive $25.**

INTEGRAL #1

READY,

GET SET,...

2:00

INTEGRAL #1

$$\int_{10}^{12} \sqrt[3]{\left(\frac{1}{2}x - 5\right)^4}\, dx$$

2:00

INTEGRAL #1

$$\int_{10}^{12} \sqrt[3]{\left(\frac{1}{2}x - 5\right)^4}\, dx$$

$$= \int_{10}^{12} \left(\frac{1}{2}x - 5\right)^{4/3} dx$$

$$= 2\int_0^1 u^{4/3}\, du \quad \left[u = \frac{1}{2}x - 5, \quad du = \frac{1}{2}dx\right]$$

$$= 2\left[\frac{3u^{7/3}}{7}\right]_0^1 = \boxed{\frac{6}{7}}$$

INTEGRAL #2

READY,

GET SET,...

2:00

INTEGRAL #2

$$\int_0^{\pi/2} (3x+2)\sin x \, dx$$

2:00

INTEGRAL #2

$$\int_0^{\pi/2} (3x+2)\sin x\,dx$$

$$\left[\text{integrate by parts: } \begin{array}{rcl} u &=& 3x+2 \\ du &=& 3\,dx \end{array}, \quad \begin{array}{rcl} dv &=& \sin x\,dx \\ v &=& -\cos x \end{array}\right]$$

$$= \Big[-(3x+2)\cos x\Big]_0^{\pi/2} + 3\int_0^{\pi/2} \cos x\,dx$$

$$= \Big[-(3x+2)\cos x\Big]_0^{\pi/2} + 3\Big[\sin x\Big]_0^{\pi/2} = \boxed{5}$$

INTEGRAL #3

READY,

GET SET,...

2:00

INTEGRAL #3

$$\int_0^2 e^x \cdot e^{2x} \cdot e^{3x} \cdot e^{4x}\,dx$$

2:00

INTEGRAL #3

$$\int_0^2 e^x \cdot e^{2x} \cdot e^{3x} \cdot e^{4x}\,dx$$

$$= \int_0^2 e^{10x}\,dx$$

$$= \left[\frac{e^{10x}}{10}\right]_0^2$$

$$= \boxed{\frac{e^{20}-1}{10}}$$

INTEGRAL #4

READY,
GET SET,...

2:00

INTEGRAL #4

$$\int_0^{\sqrt{3}} \frac{x^3}{\sqrt{x^2+1}}\,dx$$

2:00

INTEGRAL #4

$$\int_0^{\sqrt{3}} \frac{x^3}{\sqrt{x^2+1}}\,dx$$

$$= \frac{1}{2}\int_1^4 \frac{u-1}{\sqrt{u}}\,du \qquad \left[\, u = x^2+1,\; x^2 = u-1,\; 2x\,dx = du \,\right]$$

$$= \frac{1}{2}\int_1^4 \left(u^{1/2} - u^{-1/2}\right)\,du$$

$$= \frac{1}{2}\left[\frac{2u^{3/2}}{3} - 2u^{1/2}\right]_1^4 = \boxed{\frac{4}{3}}$$

INTEGRAL #5

READY,

GET SET,...

2:00

INTEGRAL #5

$$\int_0^1 \left(\sqrt{x} + 2\right)\left(5x + 3\right) \mathrm{d}x$$

2:00

INTEGRAL #5

$$\int_0^1 \left(\sqrt{x}+2\right)\left(5x+3\right)\,dx$$

$$= \int_0^1 \left(5x^{3/2}+3x^{1/2}+10x+6\right)\,dx$$

$$= \left[2x^{5/2}+2x^{3/2}+5x^2+6x\right]_0^1$$

$$= \boxed{15}$$

INTEGRAL #6

READY,

GET SET,...

2:00

INTEGRAL #6

$$\int_0^{\pi/2} \sqrt{\sin x - \sin^3 x}\, dx$$

2:00

INTEGRAL #6

$$\int_0^{\pi/2} \sqrt{\sin x - \sin^3 x}\, dx$$

$$= \int_0^{\pi/2} \sqrt{\sin x(1 - \sin^2 x)}\, dx = \int_0^{\pi/2} \sqrt{\sin x \cdot \cos^2 x}\, dx$$

$$= \int_0^{\pi/2} \sqrt{\sin x} \cdot \cos x\, dx = \int_0^1 \sqrt{u}\, du \quad [\, u = \sin x \,]$$

$$= \left[\frac{2u^{3/2}}{3}\right]_0^1 = \boxed{\frac{2}{3}}$$

INTEGRAL #7

READY,
GET SET,...

2:00

INTEGRAL #7

$$\int_0^1 \frac{x^2+3x+3}{x+1}\,dx$$

2:00

INTEGRAL #7

$$\int_0^1 \frac{x^2+3x+3}{x+1}\,dx$$

$$= \int_0^1 \left(x + 2 + \frac{1}{x+1}\right) dx \quad \text{[long division]}$$

$$= \left[\frac{x^2}{2} + 2x + \ln(x+1)\right]_0^1$$

$$= \boxed{\frac{5}{2} + \ln 2 \quad \text{or} \quad \frac{5 + 2\ln 2}{2} \quad \text{or} \quad \frac{5 + \ln 4}{2}}$$

INTEGRAL #8

READY,

GET SET,...

2:00

INTEGRAL #8

$$\int_0^{\pi} \sin x \cdot \sin \frac{x}{2}\, dx$$

2:00

INTEGRAL #8

$$\int_0^{\pi} \sin x \cdot \sin\frac{x}{2}\,dx$$

$$= \int_0^{\pi} \left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)\cdot \sin\frac{x}{2}\,dx \quad [\,\sin 2\theta = 2\sin\theta\cos\theta\,]$$

$$= 2\int_0^{\pi} \sin^2\frac{x}{2}\cos\frac{x}{2}\,dx \quad \left[u = \sin\frac{x}{2}, \quad du = \frac{1}{2}\cos\frac{x}{2}\,dx\right]$$

$$= 4\int_0^{1} u^2\,du = 4\left[\frac{u^3}{3}\right]_0^1 = \boxed{\frac{4}{3}}$$

INTEGRAL #9

READY,

GET SET,...

2:00

INTEGRAL #9

$$\int_0^1 x^{\pi} \cdot \pi^{e} \cdot x^{e} \cdot e^{\pi} \, dx$$

2:00

INTEGRAL #9

$$\int_0^1 x^{\pi} \cdot \pi^{e} \cdot x^{e} \cdot e^{\pi} \, dx$$

$$= \pi^{e} \cdot e^{\pi} \int_0^1 x^{\pi+e} \, dx$$

$$= \pi^{e} \cdot e^{\pi} \left[\frac{x^{\pi+e+1}}{\pi + e + 1} \right]_0^1$$

$$= \boxed{\frac{\pi^{e} \cdot e^{\pi}}{\pi + e + 1}}$$

INTEGRAL #10

READY,
GET SET,...

2:00

INTEGRAL #10

$$\int_0^{\pi/3} (\sin x + \tan x)(\cos x + \sec^2 x)\, dx$$

2:00

INTEGRAL #10

$$\int_0^{\pi/3} (\sin x + \tan x)(\cos x + \sec^2 x)\, dx$$

$$\left[\, u = \sin x + \tan x, \qquad du = (\cos x + \sec^2 x)\, dx \,\right]$$

$$= \int_0^{3\sqrt{3}/2} u\, du$$

$$= \left[\frac{u^2}{2}\right]_0^{3\sqrt{3}/2} = \boxed{\frac{27}{8}}$$

INTEGRAL #11

READY,

GET SET,...

2:00

INTEGRAL #11

$$\int_1^4 \frac{1}{2\sqrt{x}\sqrt{2+\sqrt{x}}}\,dx$$

2:00

INTEGRAL #11

$$\int_1^4 \frac{1}{2\sqrt{x}\sqrt{2+\sqrt{x}}}\,dx$$

$$= \int_3^4 \frac{1}{\sqrt{u}}\,du \qquad \left[u = 2+\sqrt{x}, \qquad du = \frac{1}{2\sqrt{x}}\,du \right]$$

$$= \Big[2\sqrt{u}\Big]_3^4$$

$$= \boxed{4 - 2\sqrt{3}}$$

INTEGRAL #12

READY,

GET SET,...

2:00

INTEGRAL #12

$$\int_0^{\pi/3} \sec^4 x \tan x \, dx$$

2:00

INTEGRAL #12

$$\int_0^{\pi/3} \sec^4 x \tan x \, dx$$

$$= \int_0^{\pi/3} \sec^3 x \cdot \sec x \tan x \, dx$$

$$= \int_1^2 u^3 \, du \quad [\, u = \sec x, \quad du = \sec x \tan x \, dx \,]$$

$$= \left[\frac{u^4}{4}\right]_1^2 = \boxed{\frac{15}{4}}$$

INTEGRAL #13

READY,
GET SET,...

2:00

INTEGRAL #13

$$\int_1^2 \frac{xe^{2x} + \ln x}{x}\,dx$$

2:00

INTEGRAL #13

$$\int_1^2 \frac{xe^{2x} + \ln x}{x}\, dx$$

$$= \int_1^2 \left(\frac{xe^{2x}}{x} + \frac{\ln x}{x} \right) dx$$

$$= \int_1^2 \left(e^{2x} + \frac{\ln x}{x} \right) dx$$

$$= \left[\frac{e^{2x}}{2} + \frac{(\ln x)^2}{2} \right]_1^2 = \boxed{\frac{e^4 - e^2 + (\ln 2)^2}{2}}$$

INTEGRAL #14

READY,

GET SET,...

2:00

INTEGRAL #14

$$\int_0^1 \arctan x \, dx$$

2:00

INTEGRAL #14

$$\int_0^1 \arctan x \, dx$$

$$\left[\text{integrate by parts: } \begin{array}{rcl} u &=& \arctan x \\ du &=& \frac{1}{x^2+1}\,dx \end{array}, \quad \begin{array}{rcl} dv &=& dx \\ v &=& x \end{array} \right]$$

$$= \Big[x \arctan x\Big]_0^1 - \int_0^1 \frac{x}{x^2+1}\,dx = \frac{\pi}{4} - \left[\frac{1}{2}\ln\left(x^2+1\right)\right]_0^1$$

$$= \boxed{\frac{\pi}{4} - \frac{\ln 2}{2} \quad \text{or} \quad \frac{\pi - 2\ln 2}{4} \quad \text{or} \quad \frac{\pi - \ln 4}{4}}$$

INTEGRAL #15

READY,

GET SET,...

2:00

INTEGRAL #15

$$\int_1^7 \frac{2014}{\dfrac{20}{\frac{20}{x}+\frac{14}{x}}+\dfrac{14}{\frac{20}{x}+\frac{14}{x}}}\,dx$$

2:00

INTEGRAL #15

$$\int_1^7 \frac{2014}{\dfrac{20}{\frac{20}{x}+\frac{14}{x}}+\dfrac{14}{\frac{20}{x}+\frac{14}{x}}}\,dx$$

$$= \int_1^7 \frac{2014}{x}\,dx \quad \text{[Simplify!]}$$

$$= 2014\Big[\ln x\Big]_1^7$$

$$= \boxed{2014 \ln 7}$$