THE HIGH SCHOOL FINALS

The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,

GET SET,...

2:30

INTEGRAL #1

$$\int \frac{1}{x^3}\left(\frac{x^2+1}{x^2}\right)^9 dx$$

2:30

INTEGRAL #1

$$\int \frac{1}{x^3}\left(\frac{x^2+1}{x^2}\right)^9 dx$$

$$= \int \frac{1}{x^3}\left(1+\frac{1}{x^2}\right)^9 dx \quad \left[u = 1+\frac{1}{x^2}, \quad du = -\frac{2}{x^3}\right]$$

$$= -\frac{1}{2}\int u^9\, du = -\frac{1}{2}\cdot\frac{u^{10}}{10} + C$$

$$= \boxed{-\frac{1}{20}\left(1+\frac{1}{x^2}\right)^{10}}$$

INTEGRAL #2

READY,

GET SET,...

2:30

INTEGRAL #2

$$\int \frac{\sin x}{\cos^4 x}\,dx$$

2:30

INTEGRAL #2

$$\int \frac{\sin x}{\cos^4 x}\,dx$$

$$= -\int \frac{1}{u^4}\,du \qquad [\, u = \cos x, \quad du = -\sin x \,]$$

$$= \frac{1}{3u^3} + C$$

$$= \boxed{\frac{1}{3\cos^3 x} + C}$$

INTEGRAL #3

READY,

GET SET,...

2:30

INTEGRAL #3

$$\int \frac{4}{4x^2+4x+1}\,dx$$

2:30

INTEGRAL #3

$$\int \frac{4}{4x^2+4x+1}\,dx$$

$$= \int \frac{4}{(2x+1)^2}\,dx \qquad [\, u = 2x+1, \quad du = 2\,dx \,]$$

$$= \int \frac{2}{u^2}\,du$$

$$= -\frac{2}{u} + C = \boxed{-\frac{2}{2x+1} + C}$$

INTEGRAL #4

READY,

GET SET,...

2:30

INTEGRAL #4

$$\int \frac{e^x + 2e^{2x} + 3e^{3x}}{4e^{4x}}\,dx$$

2:30

INTEGRAL #4

$$\int \frac{e^x + 2e^{2x} + 3e^{3x}}{4e^{4x}}\,dx$$

$$= \int \left(\frac{e^x}{4e^{4x}} + \frac{2e^{2x}}{4e^{4x}} + \frac{3e^{3x}}{4e^{4x}} \right) dx$$

$$= \int \left(\frac{1}{4}e^{-3x} + \frac{1}{2}e^{-2x} + \frac{3}{4}e^{-x} \right) dx$$

$$= \boxed{-\frac{e^{-3x}}{12} - \frac{e^{-2x}}{4} - \frac{3e^{-x}}{4} + C}$$

INTEGRAL #5

READY,

GET SET,...

2:30

INTEGRAL #5

$$\int \frac{3\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$$

2:30

INTEGRAL #5

$$\int \frac{3\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$$

$$= 6\int \sqrt{u}\,du \quad \left[u = 1+\sqrt{x}, \quad du = \frac{1}{2\sqrt{x}}\,dx\right]$$

$$= 4u^{3/2} + C$$

$$= \boxed{4(1+\sqrt{x})^{3/2} + C}$$

INTEGRAL #6

READY,

GET SET,...

2:30

INTEGRAL #6

$$\int e^x \sin e^x \cos e^x \, dx$$

2:30

INTEGRAL #6

$$\int e^x \sin e^x \cos e^x \, dx$$

$$= \int u \, du \quad [\, u = \sin e^x, \quad du = e^x \cos e^x \, dx \,]$$

$$= \frac{u^2}{2} + C = \boxed{\frac{\sin^2 e^x}{2} + C \quad \text{or} \quad -\frac{\cos^2 e^x}{2} + C}$$

INTEGRAL #7

READY,

GET SET,...

2:30

INTEGRAL #7

$$\int \frac{\cot x - \sec x}{\cot x}\, dx$$

2:30

INTEGRAL #7

$$\int \frac{\cot x - \sec x}{\cot x}\, dx$$

$$= \int \left(\frac{\cot x}{\cot x} - \frac{\sec x}{\cot x} \right)\, dx$$

$$= \int (1 - \sec x \tan x)\, dx$$

$$= \boxed{x - \sec x + C}$$

INTEGRAL #8

READY,

GET SET,...

2:30

INTEGRAL #8

$$\int \frac{(x+1)^3 + (x+2)^3}{(x+1)^3(x+2)^3}\,dx$$

2:30

INTEGRAL #8

$$\int \frac{(x+1)^3 + (x+2)^3}{(x+1)^3(x+2)^3}\,dx$$

$$= \int \left(\frac{(x+1)^3}{(x+1)^3(x+2)^3} + \frac{(x+2)^3}{(x+1)^3(x+2)^3} \right) dx$$

$$= \int \left(\frac{1}{(x+2)^3} + \frac{1}{(x+1)^3} \right) dx$$

$$= \boxed{-\frac{1}{2(x+2)^2} - \frac{1}{2(x+1)^2} + C}$$

INTEGRAL #9

READY,

GET SET,...

2:30

INTEGRAL #9

$$\int \frac{\sqrt{\tan x}}{1 - \sin^2 x}\, dx$$

2:30

INTEGRAL #9

$$\int \frac{\sqrt{\tan x}}{1 - \sin^2 x}\, dx$$

$$= \int \frac{\sqrt{\tan x}}{\cos^2 x}\, dx = \int \sqrt{\tan x}\, \sec^2 x\, dx$$

$$= \int \sqrt{u}\, du \qquad \left[u = \tan x, \quad du = \sec^2 x\, dx\right]$$

$$= \frac{2u^{3/2}}{3} + C = \boxed{\frac{2\tan^{3/2} x}{3} + C}$$

INTEGRAL #10

READY,

GET SET,...

2:30

INTEGRAL #10

$$\int \frac{\sqrt[3]{x}}{x\sqrt[3]{33+\sqrt[3]{x}}}\,dx$$

2:30

INTEGRAL #10

$$\int \frac{\sqrt[3]{x}}{x\sqrt[3]{33+\sqrt[3]{x}}}\,dx$$

$$= \int \frac{x^{-2/3}}{\sqrt[3]{33+\sqrt[3]{x}}}\,dx$$

$$= \int \frac{3}{\sqrt[3]{u}}\,du \qquad \left[u = 33+\sqrt[3]{x}, \quad du = \frac{1}{3}x^{-2/3}\,dx \right]$$

$$= \frac{9u^{2/3}}{2} + C = \boxed{\frac{9(33+\sqrt[3]{x})^{2/3}}{2} + C}$$