THE COLLEGE FINALS

The finals are conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,
GET SET,...

2:30

INTEGRAL #1

$$\int \sin^2 x \cdot \sin 2x \, dx$$

2:30

INTEGRAL #1

$$\int \sin^2 x \cdot \sin 2x \, dx$$

$$= \int \sin^2 x \cdot 2 \sin x \cos x \, dx = 2 \int \sin^3 x \cdot \cos x \, dx$$

$$= 2 \int u^3 \, du \quad [\, u = \sin x, \quad du = \cos x \, dx \,]$$

$$= \boxed{\frac{\sin^4 x}{2} + C}$$

INTEGRAL #2

READY,
GET SET,...

2:30

INTEGRAL #2

$$\int \frac{\pi x - \arctan x}{x^2 + 1}\, dx$$

2:30

INTEGRAL #2

$$\int \frac{\pi x - \arctan x}{x^2+1}\,dx$$

$$= \int \left(\frac{\pi x}{x^2+1} - \frac{\arctan x}{x^2+1} \right) dx$$

$$= \boxed{\frac{\pi}{2}\ln(x^2+1) - \frac{1}{2}(\arctan x)^2 + C}$$

INTEGRAL #3

READY,
GET SET,...

2:30

INTEGRAL #3

$$\int \frac{x}{x^2+3x+2}\,dx$$

2:30

INTEGRAL #3

$$\int \frac{x}{x^2+3x+2}\,dx$$

$$= \int \frac{x}{(x+2)(x+1)}\,dx$$

$$= \int \left(\frac{2}{x+2} - \frac{1}{x+1}\right)\,dx$$

$$= \boxed{2\ln(x+2) - \ln(x+1) + C}$$

INTEGRAL #4

READY,
GET SET,...

2:30

INTEGRAL #4

$$\int \frac{\sin x - \cos x}{\sin x + \cos x}\, dx$$

2:30

INTEGRAL #4

$$\int \frac{\sin x - \cos x}{\sin x + \cos x}\,dx$$

$$= -\int \frac{1}{u}\,du \qquad \left[\, u = \sin x + \cos x, \qquad du = (\cos x - \sin x)\,dx \,\right]$$

$$= -\ln u + C$$

$$= \boxed{-\ln|\sin x + \cos x| + C}$$

INTEGRAL #5

READY,
GET SET,...

2:30

INTEGRAL #5

$$\int \arctan \frac{1}{x}\, dx$$

2:30

INTEGRAL #5

$$\int \arctan\frac{1}{x}\,dx$$

$$\left[\text{integrate by parts: } \begin{array}{cc} u=\arctan\frac{1}{x} & dv=dx \\ du=-\frac{1}{x^2+1}\,dx, & v=x \end{array}\right]$$

$$= x\arctan\frac{1}{x}+\int\frac{x}{x^2+1}\,dx$$

$$= \boxed{x\arctan\frac{1}{x}+\frac{1}{2}\ln(x^2+1)+C}$$