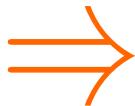


THE COLLEGE FINALS



The finals are conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional \$75 and will be crowned the **Integration Champion!**

INTEGRAL #1

**READY,
GET SET,...**

2:30

INTEGRAL #1

$$\int \sin^2 x \cdot \sin 2x \, dx$$

INTEGRAL #1

$$\int \sin^2 x \cdot \sin 2x \, dx$$

$$= \int \sin^2 x \cdot 2 \sin x \cos x \, dx = 2 \int \sin^3 x \cdot \cos x \, dx$$

$$= 2 \int u^3 \, du \quad [u = \sin x, \quad du = \cos x \, dx]$$

$$= \boxed{\frac{\sin^4 x}{2} + C}$$

INTEGRAL #2

**READY,
GET SET,...**

2:30

INTEGRAL #2

$$\int \frac{\pi x - \arctan x}{x^2 + 1} dx$$

INTEGRAL #2

$$\int \frac{\pi x - \arctan x}{x^2 + 1} dx$$

$$= \int \left(\frac{\pi x}{x^2 + 1} - \frac{\arctan x}{x^2 + 1} \right) dx$$

$$= \boxed{\frac{\pi}{2} \ln(x^2 + 1) - \frac{1}{2}(\arctan x)^2 + C}$$

INTEGRAL #3

**READY,
GET SET,...**

2:30

INTEGRAL #3

$$\int \frac{x}{x^2 + 3x + 2} dx$$

INTEGRAL #3

$$\int \frac{x}{x^2 + 3x + 2} dx$$

$$= \int \frac{x}{(x+2)(x+1)} dx$$

$$= \int \left(\frac{2}{x+2} - \frac{1}{x+1} \right) dx$$

$$= \boxed{2 \ln(x+2) - \ln(x+1) + C}$$

INTEGRAL #4

**READY,
GET SET,...**

2:30

INTEGRAL #4

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

INTEGRAL #4

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

$$= - \int \frac{1}{u} du \quad [u = \sin x + \cos x, \quad du = (\cos x - \sin x) dx]$$

$$= -\ln u + C$$

$$= -\ln|\sin x + \cos x| + C$$

INTEGRAL #5

**READY,
GET SET,...**

2:30

INTEGRAL #5

$$\int \arctan \frac{1}{x} dx$$

INTEGRAL #5

$$\int \arctan \frac{1}{x} dx$$

[integrate by parts: $u = \arctan \frac{1}{x}$, $dv = dx$
 $du = -\frac{1}{x^2+1} dx$, $v = x$]

$$= x \arctan \frac{1}{x} + \int \frac{x}{x^2+1} dx$$

$$= x \arctan \frac{1}{x} + \frac{1}{2} \ln(x^2+1) + C$$