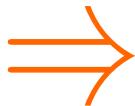


THE HIGH SCHOOL FINALS



The finals are conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional \$75 and will be crowned the **Integration Champion!**

INTEGRAL #1

**READY,
GET SET,...**

2:30

INTEGRAL #1

$$\int (x^2 + 1)(x^3 + 3x)^4 \, dx$$

INTEGRAL #1

$$\int (x^2 + 1)(x^3 + 3x)^4 dx$$

$$[u = x^3 + 3x, \quad du = (3x^2 + 3) dx = 3(x^2 + 1) dx]$$

$$= \frac{1}{3} \int u^4 du$$

$$= \frac{u^5}{15} + C = \boxed{\frac{(x^3 + 3x)^5}{15} + C}$$

INTEGRAL #2

**READY,
GET SET,...**

2:30

INTEGRAL #2

$$\int \frac{1}{\cos^2 x \sqrt{1 + \tan x}} dx$$

INTEGRAL #2

$$\int \frac{1}{\cos^2 x \sqrt{1 + \tan x}} dx$$

$$= \int \frac{\sec^2 x}{\sqrt{1 + \tan x}} dx$$

$$= \int \frac{1}{\sqrt{u}} du \quad [u = 1 + \tan x, \quad du = \sec^2 x dx]$$

$$= 2\sqrt{u} + C = \boxed{2\sqrt{1 + \tan x} + C}$$

INTEGRAL #3

**READY,
GET SET,...**

2:30

INTEGRAL #3

$$\int \left(\frac{2 - x^2}{x^2} \right)^2 dx$$

INTEGRAL #3

$$\int \left(\frac{2 - x^2}{x^2} \right)^2 dx$$

$$= \int \left(\frac{2}{x^2} - 1 \right)^2 dx$$

$$= \int \left(\frac{4}{x^4} - \frac{4}{x^2} + 1 \right) dx$$

$$= \boxed{-\frac{4}{3x^3} + \frac{4}{x} + x + C}$$

INTEGRAL #4

**READY,
GET SET,...**

2:30

INTEGRAL #4

$$\int x \cos^2(x^2) \sin(x^2) dx$$

INTEGRAL #4

$$\int x \cos^2(x^2) \sin(x^2) dx$$

$$= -\frac{1}{2} \int u^2 du \quad [u = \cos(x^2), \quad du = -2x \sin(x^2) dx]$$

$$= -\frac{u^3}{6} + C$$

$$= -\frac{\cos^3(x^2)}{6} + C$$

INTEGRAL #5

**READY,
GET SET,...**

2:30

INTEGRAL #5

$$\int (x + 1)(e^x + 1) dx$$

INTEGRAL #5

$$\int (x+1)(e^x + 1) dx$$

[integrate by parts: $u = x + 1$, $dv = (e^x + 1) dx$
 $du = dx$, $v = e^x + x$]

$$= (x+1)(e^x + x) - \int (e^x + x) dx$$

$$= (x+1)(e^x + x) - e^x - \frac{x^2}{2} + C = xe^x + \frac{x^2}{2} + x + C$$

INTEGRAL #6

**READY,
GET SET,...**

2:30

INTEGRAL #6

$$\int (x^2 + \sin 2x)(x + \cos 2x) \, dx$$

INTEGRAL #6

$$\int (x^2 + \sin 2x)(x + \cos 2x) dx$$

$$= \frac{1}{2} \int u du \quad [u = x^2 + \sin 2x, \quad du = (2x + 2 \cos 2x) dx]$$

$$= \frac{u^2}{4} + C$$

$$= \frac{(x^2 + \sin 2x)^2}{4} + C$$

INTEGRAL #7

**READY,
GET SET,...**

2:30

INTEGRAL #7

$$\int x^2 \sqrt{x+2} \, dx$$

INTEGRAL #7

$$\int x^2 \sqrt{x+2} dx$$

$$= \int (u - 2)^2 \sqrt{u} du \quad [u = x + 2, \quad du = dx]$$

$$= \int (u^{5/2} - 4u^{3/2} + 4u^{1/2}) du$$

$$= \boxed{\frac{2(x+2)^{7/2}}{7} - \frac{8(x+2)^{5/2}}{5} + \frac{8(x+2)^{3/2}}{3} + C}$$

INTEGRAL #8

**READY,
GET SET,...**

2:30

INTEGRAL #8

$$\int \frac{4x^2 + 4x + 5}{4x^2 + 4x + 1} dx$$

INTEGRAL #8

$$\int \frac{4x^2 + 4x + 5}{4x^2 + 4x + 1} dx$$

$$= \int \left(\frac{4x^2 + 4x + 1}{4x^2 + 4x + 1} + \frac{4}{4x^2 + 4x + 1} \right) dx$$

$$= \int \left(1 + \frac{4}{(2x+1)^2} \right) dx$$

$$= x - \frac{2}{2x+1} + C$$

INTEGRAL #9

**READY,
GET SET,...**

2:30

INTEGRAL #9

$$\int \sqrt{1 + x^{-2/3}} dx$$

INTEGRAL #9

$$\int \sqrt{1 + x^{-2/3}} dx$$

$$= \int \sqrt{1 + \frac{1}{x^{2/3}}} dx = \int \sqrt{\frac{x^{2/3} + 1}{x^{2/3}}} dx = \int \frac{\sqrt{x^{2/3} + 1}}{x^{1/3}} dx$$

$$= \frac{3}{2} \int \sqrt{u} du \quad \left[u = x^{2/3} + 1, \quad du = \frac{2}{3} x^{-1/3} dx \right]$$

$$= (x^{2/3} + 1)^{3/2} + C$$