THE COLLEGE FINALS

The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,
GET SET,...

2:30

INTEGRAL #1

$$\int \frac{x}{x^4 + 4x^2 + 4}\, dx$$

INTEGRAL #1

$$\int \frac{x}{x^4+4x^2+4}\,dx$$

$$= \int \frac{x}{(x^2+2)^2}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u^2}\,du \qquad u = x^2+2,\ du = 2x\,dx$$

$$= -\frac{1}{2u} + C = \boxed{-\frac{1}{2(x^2+2)} + C}$$

INTEGRAL #2

READY,
GET SET,...

2:30

INTEGRAL #2

$$\int \sin^3 x \cos^2 x \, dx$$

INTEGRAL #2

$$\int \sin^3 x \cos^2 x \, dx$$

$$= \int (1 - \cos^2 x) \cos^2 x \sin x \, dx$$

$$= -\int (1 - u^2) u^2 \, du \qquad u = \cos x, \quad du = -\sin x \, dx$$

$$= -\int (u^2 - u^4) \, du = \boxed{-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C}$$

INTEGRAL #3

READY,
GET SET,...

2:30

INTEGRAL #3

$$\int \frac{(x+2)^3}{x}\,dx$$

INTEGRAL #3

$$\int \frac{(x+2)^3}{x}\,dx$$

$$= \int \frac{x^3+6x^2+12x+8}{x}\,dx$$

$$= \int \left(x^2+6x+12+\frac{8}{x}\right)\,dx$$

$$= \boxed{\frac{x^3}{3}+3x^2+12x+8\ln|x|+C}$$

INTEGRAL #4

READY,
GET SET,...

2:30

INTEGRAL #4

$$\int \frac{e^x + e^{2x} + e^{3x}}{e^{4x}}\,dx$$

INTEGRAL #4

$$\int \frac{e^x + e^{2x} + e^{3x}}{e^{4x}}\,dx$$

$$= \int \left(\frac{e^x}{e^{4x}} + \frac{e^{2x}}{e^{4x}} + \frac{e^{3x}}{e^{4x}}\right)\,dx$$

$$= \int \left(e^{-3x} + e^{-2x} + e^{-x}\right)\,dx$$

$$= \boxed{-\frac{1}{3}e^{-3x} - \frac{1}{2}e^{-2x} - e^{-x} + C}$$

INTEGRAL #5

READY,
GET SET,...

2:30

INTEGRAL #5

$$\int \sqrt{x} \ln x \, dx$$

INTEGRAL #5

$$\int \sqrt{x} \ln x \, dx$$

$$= \frac{2x^{3/2} \ln x}{3} - \frac{2}{3} \int x^{1/2} \, dx$$

integrate by parts

$$= \boxed{\frac{2x^{3/2} \ln x}{3} - \frac{4x^{3/2}}{9} + C}$$

INTEGRAL #6

READY,
GET SET,...

2:30

INTEGRAL #6

$$\int \frac{\sec^2 2x}{\sqrt[3]{2+\tan 2x}}\,dx$$

INTEGRAL #6

$$\int \frac{\sec^2 2x}{\sqrt[3]{2+\tan 2x}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt[3]{u}}\,du \qquad u = 2+\tan 2x,\ \ du = 2\sec^2 2x\,dx$$

$$= \frac{3u^{2/3}}{4} + C$$

$$= \boxed{\frac{3(2+\tan 2x)^{2/3}}{4} + C}$$

INTEGRAL #7

READY,
GET SET,...

2:30

INTEGRAL #7

$$\int (x+17)\sqrt{x+29}\,dx$$

INTEGRAL #7

$$\int (x+17)\sqrt{x+29}\,dx$$

$$= \int (u-12)\sqrt{u}\,du \qquad u = x+29,\ x = u-29,\ dx = du$$

$$= \int \left(u^{3/2} - 12u^{1/2}\right)\,du = \frac{2u^{5/2}}{5} - 8u^{3/2} + C$$

$$= \boxed{\frac{2}{5}(x+29)^{5/2} - 8(x+29)^{3/2} = \frac{2}{5}(x+9)(x+29)^{3/2} + C}$$

INTEGRAL #8

READY,
GET SET,...

2:30

INTEGRAL #8

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx$$

INTEGRAL #8

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx$$

$$= \int \left(\frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} \right) dx$$

$$= \boxed{\arcsin x + \sqrt{1-x^2} + C}$$

INTEGRAL #9

READY,
GET SET,...

2:30

INTEGRAL #9

$$\int \frac{\sin x}{(1-\sin x)(1+\sin x)}\,dx$$

INTEGRAL #9

$$\int \frac{\sin x}{(1-\sin x)(1+\sin x)}\,dx$$

$$= \int \frac{\sin x}{1-\sin^2 x}\,dx = \int \frac{\sin x}{\cos^2 x}\,dx$$

$$= -\int \frac{1}{u^2} \qquad u = \cos x, \;\; du = -\sin x\,dx$$

$$= \frac{1}{u} + C = \boxed{\frac{1}{\cos x} + C = \sec x + C}$$

INTEGRAL #10

READY,
GET SET,...

2:30

INTEGRAL #10

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$$

INTEGRAL #10

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$$

$$= \int \frac{\cos\theta}{\sin^2\theta \cdot \cos\theta}\,d\theta \qquad x = \sin\theta,\ \ dx = \cos\theta\,d\theta$$

$$= \int \csc^2\theta\,d\theta = -\cot\theta + C$$

$$= \boxed{-\frac{\sqrt{1-x^2}}{x} + C}$$

INTEGRAL #11

READY,
GET SET,...

2:30

INTEGRAL #11

$$\int \frac{x-1}{x^2(x+1)}\,dx$$

INTEGRAL #11

$$\int \frac{x-1}{x^2(x+1)}\,dx$$

$$= \int \left(\frac{2}{x} - \frac{1}{x^2} - \frac{2}{x+1}\right) dx$$ partial fractions

$$= \boxed{2\ln|x| + \frac{1}{x} - 2\ln|x+1| + C}$$

INTEGRAL #12

READY,
GET SET,...

2:30

INTEGRAL #12

$$\int \frac{\sin x + \sec x}{\tan x}\, dx$$

INTEGRAL #12

$$\int \frac{\sin x + \sec x}{\tan x}\,dx$$

$$= \int \left(\frac{\sin x}{\tan x} + \frac{\sec x}{\tan x}\right)\,dx$$

$$= \int (\cos x + \csc x)\,dx$$

$$= \boxed{\sin x + \ln|\csc x - \cot x| + C}$$

INTEGRAL #13

READY,
GET SET,...

2:30

INTEGRAL #13

$$\int \frac{\ln x}{x + x(\ln x)^2}\, dx$$

INTEGRAL #13

$$\int \frac{\ln x}{x + x(\ln x)^2}\,dx$$

$$= \int \frac{\ln x}{x(1 + (\ln x)^2)}\,dx$$

$$= \int \frac{u}{1 + u^2} \qquad u = \ln x,\ \ du = \frac{1}{x}\,dx$$

$$= \frac{1}{2}\ln(1 + u^2) + C = \boxed{\frac{1}{2}\ln(1 + (\ln x)^2) + C}$$