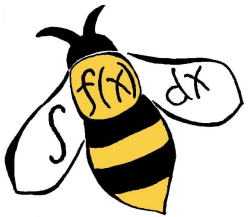
# THE COLLEGE FINALS




The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

**The last person remaining wins an additional $75** and will be crowned the **Integration Champion**!

INTEGRAL #1

# READY,

# GET SET,...

# 2:30

**INTEGRAL #1**

$$\int (x^3 + 2)^3 \, dx$$

**INTEGRAL #1**

$$\int (x^3 + 2)^3 \, dx$$

$$= \int \left(x^9 + 6x^6 + 12x^3 + 8\right) \, dx$$

$$= \boxed{\frac{x^{10}}{10} + \frac{6x^7}{7} + 3x^4 + 8x + C}$$

INTEGRAL #2

# READY,

# GET SET,...

# 2:30

**INTEGRAL #2**

$$\int \frac{\sqrt[3]{x} + \sqrt[4]{x}}{\sqrt[5]{x}}\,dx$$

**INTEGRAL #2**

$$\int \frac{\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt[5]{x}}\,dx$$

$$= \int \frac{x^{1/3}+x^{1/4}}{x^{1/5}}\,dx$$

$$= \int \left(\frac{x^{1/3}}{x^{1/5}}+\frac{x^{1/4}}{x^{1/5}}\right)\,dx$$

$$= \int \left(x^{2/15}+x^{1/20}\right)\,dx = \boxed{\frac{15x^{17/15}}{17}+\frac{20x^{21/20}}{21}+C}$$

INTEGRAL #3

# READY,

# GET SET,...

# 2:30

**INTEGRAL #3**

$$\int \left(\sqrt{x}+1\right)\left(x-\sqrt{x}+1\right)\,dx$$

**INTEGRAL #3**

$$\int \left(\sqrt{x}+1\right)\left(x-\sqrt{x}+1\right)\,dx$$

$$= \int \left(x^{3/2}+1\right)\,dx$$

$$= \boxed{\frac{2x^{5/2}}{5}+x+C}$$

INTEGRAL #4

# READY,

# GET SET,...

# 2:30

**INTEGRAL #4**

$$\int \left(\frac{x-5}{x}\right)^2 dx$$

**INTEGRAL #4**

$$\int \left(\frac{x-5}{x}\right)^2 dx$$

$$= \int \frac{x^2 - 10x + 25}{x^2} dx$$

$$= \int \left(1 - \frac{10}{x} + \frac{25}{x^2}\right) dx$$

$$= \boxed{x - 10\ln|x| - \frac{25}{x} + C}$$

INTEGRAL #5

# READY,

# GET SET,...

# 2:30

**INTEGRAL #5**

$$\int x^{e} \cdot e^{\pi} \cdot x^{\pi} \, dx$$

**INTEGRAL #5**

$$\int x^e \cdot e^\pi \cdot x^\pi \, dx$$

$$= e^\pi \int x^{\pi+e} \, dx$$

$$= \boxed{\frac{e^\pi x^{\pi+e+1}}{\pi+e+1} + C}$$

INTEGRAL #6

# READY,

# GET SET,...

# 2:30

**INTEGRAL #6**

$$\int \sec^7 x \tan x \, dx$$

**INTEGRAL #6**

$$\int \sec^7 x \tan x \, dx$$

$$= \int \sec^6 x \cdot \sec x \tan x \, dx$$

$$= \int u^6 \, du \qquad u = \sec x, \quad du = \sec x \tan x \, dx$$

$$= \frac{u^7}{7} + C = \boxed{\frac{\sec^7 x}{7} + C \text{ or } \frac{1}{7\cos^7 x} + C}$$

INTEGRAL #7

# READY,

# GET SET,...

# 2:30

**INTEGRAL #7**

$$\int \frac{x}{e^{3x}}\,dx$$

**INTEGRAL #7**

$$\int \frac{x}{e^{3x}}\,dx$$

$$= \int xe^{-3x}\,dx =$$

$$-\frac{xe^{-3x}}{3} + \frac{1}{3}\int e^{-3x}\,dx$$ integration by parts

$$= \boxed{-\frac{xe^{-3x}}{3} - \frac{e^{-3x}}{9} + C}$$

INTEGRAL #8

# READY,

# GET SET,...

# 2:30

**INTEGRAL #8**

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx$$

**INTEGRAL #8**

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

$$= 2\int \sin u\,du \qquad u = \sqrt{x}, \quad du = \frac{1}{2\sqrt{x}}\,dx$$

$$= -2\cos u + C$$

$$= \boxed{-2\cos\sqrt{x} + C}$$

INTEGRAL #9

# READY,

# GET SET,...

# 2:30

**INTEGRAL #9**

$$\int \frac{1}{\cos^2 x(1+\tan x)^2}\,dx$$

**INTEGRAL #9**

$$\int \frac{1}{\cos^2 x(1+\tan x)^2}\,dx$$

$$= \int \frac{\sec^2 x}{(1+\tan x)^2}\,dx$$

$$= \int \frac{1}{u^2}\,du \qquad u = 1+\tan x, \quad , \; du = \sec^2 x\,dx$$

$$= -\frac{1}{u} + C = \boxed{-\frac{1}{1+\tan x} + C}$$

INTEGRAL #10

# READY,

# GET SET,...

# 2:30

**INTEGRAL #10**

$$\int (3 + \sin 5x)(3 - \sin 5x)\, dx$$

**INTEGRAL #10**

$$\int (3+\sin 5x)(3-\sin 5x)\,dx$$

$$= \int \left(9-\sin^2 5x\right)\,dx$$

$$= 9x - \int \frac{1-\cos 10x}{2}\,dx$$

$$= 9x - \frac{x}{2} + \frac{\sin 10x}{20} + C = \boxed{\frac{17x}{2} + \frac{\sin 10x}{20} + C}$$

INTEGRAL #11

# READY,

# GET SET,...

# 2:30

**INTEGRAL #11**

$$\int \frac{1}{5-\sqrt{x}}\,dx$$

**INTEGRAL #11**

$$\int \frac{1}{5-\sqrt{x}}\,dx$$

$$= -2\int \frac{5-u}{u}\,du \qquad u = 5-\sqrt{x}, \quad du = -\frac{1}{2\sqrt{x}}\,dx$$

$$= -2\left(5\ln|u| - u\right) + C$$

$$= \boxed{-2\left(5\ln|5-\sqrt{x}| - 5 + \sqrt{x}\right) + C}$$

INTEGRAL #12

# READY,

# GET SET,...

# 2:30

**INTEGRAL #12**

$$\int (\sec x - \tan x)^2 \, dx$$

**INTEGRAL #12**

$$\int (\sec x - \tan x)^2 \, dx$$

$$= \int (\sec^2 - 2\sec x \tan x + \tan^2 x) \, dx$$

$$= \int (\sec^2 - 2\sec x \tan x + \sec^2 x - 1) \, dx$$

$$= \int (2\sec^2 - 2\sec x \tan x - 1) \, dx$$

$$= \boxed{2\tan x - 2\sec x - x + C}$$

INTEGRAL #13

# READY,

# GET SET,...

# 2:30

**INTEGRAL #13**

$$\int \frac{2}{(x+1)(x+2)(x+3)}\,dx$$

**INTEGRAL #13**

$$\int \frac{2}{(x+1)(x+2)(x+3)}\,dx$$

$$= \int \left(\frac{1}{x+1} - \frac{2}{x+2} + \frac{1}{x+3}\right) dx$$

partial fractions

$$= \boxed{\ln|x+1| - 2\ln|x+2| + \ln|x+3| + C}$$