

THE HIGH SCHOOL FINALS



The finals are conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional \$75 and will be crowned the Integration Champion!

INTEGRAL #1

**READY,
GET SET,...**

2:30

INTEGRAL #1

$$\int (x^3 + 2)^3 dx$$

INTEGRAL #1

$$\int (x^3 + 2)^3 dx$$

$$= \int (x^9 + 6x^6 + 12x^3 + 8) dx$$

$$= \frac{x^{10}}{10} + \frac{6x^7}{7} + 3x^4 + 8x + C$$

INTEGRAL #2

**READY,
GET SET,...**

2:30

INTEGRAL #2

$$\int (\sqrt{x} + 1) (x - \sqrt{x} + 1) dx$$

INTEGRAL #2

$$\int (\sqrt{x} + 1) (x - \sqrt{x} + 1) dx$$

$$= \int (x^{3/2} + 1) dx$$

$$= \frac{2x^{5/2}}{5} + x + C$$

INTEGRAL #3

**READY,
GET SET,...**

2:30

INTEGRAL #3

$$\int \left(\frac{x-5}{x} \right)^2 dx$$

INTEGRAL #3

$$\begin{aligned} & \int \left(\frac{x-5}{x} \right)^2 dx \\ &= \int \frac{x^2 - 10x + 25}{x^2} dx \\ &= \int \left(1 - \frac{10}{x} + \frac{25}{x^2} \right) dx \\ &= x - 10 \ln|x| - \frac{25}{x} + C \end{aligned}$$

INTEGRAL #4

**READY,
GET SET,...**

2:30

INTEGRAL #4

$$\int x^e \cdot e^\pi \cdot x^\pi dx$$

INTEGRAL #4

$$\int x^e \cdot e^\pi \cdot x^\pi dx$$

$$= e^\pi \int x^{\pi+e} dx$$

$$= \frac{e^\pi x^{\pi+e+1}}{\pi+e+1} + C$$

INTEGRAL #5

**READY,
GET SET,...**

2:30

INTEGRAL #5

$$\int \left(\frac{2}{\sqrt{x}} + \sqrt{x} \right)^2 dx$$

INTEGRAL #5

$$\int \left(\frac{2}{\sqrt{x}} + \sqrt{x} \right)^2 dx$$

$$= \int \left(\frac{4}{x} + 4 + x \right) dx$$

$$= 4 \ln|x| + 4x + \frac{x^2}{2} + C$$

INTEGRAL #6

**READY,
GET SET,...**

2:30

INTEGRAL #6

$$\int \frac{\sqrt{5 - \sqrt{x}}}{\sqrt{x}} dx$$

INTEGRAL #6

$$\int \frac{\sqrt{5 - \sqrt{x}}}{\sqrt{x}} dx$$

$$= -2 \int \sqrt{u} du \quad u = 5 - \sqrt{x}, \quad du = -\frac{1}{2\sqrt{x}} dx$$

$$= -\frac{4}{3} u^{3/2} + C$$

$$= \frac{4(5 - \sqrt{x})^{3/2}}{3} + C$$

INTEGRAL #7

**READY,
GET SET,...**

2:30

INTEGRAL #7

$$\int (\sec x \tan x)^2 dx$$

INTEGRAL #7

$$\int (\sec x \tan x)^2 dx$$

$$= \int \sec^2 x \tan^2 x dx$$

$$= \int u^2 du \quad u = \tan x, \quad du = \sec^2 x dx$$

$$= \frac{\tan^3}{3} + C$$

INTEGRAL #8

**READY,
GET SET,...**

2:30

INTEGRAL #8

$$\int \frac{1}{x\sqrt{x}} \left(1 + \frac{1}{\sqrt{x}} \right)^7 dx$$

INTEGRAL #8

$$\int \frac{1}{x\sqrt{x}} \left(1 + \frac{1}{\sqrt{x}}\right)^7 dx$$

$$= -2 \int u^7 du \quad u = 1 + \frac{1}{\sqrt{x}}, \quad du = -\frac{1}{2x\sqrt{x}} dx$$

$$= -\frac{u^8}{8} + C$$

$$= -\frac{1}{8} \left(1 + \frac{1}{\sqrt{x}}\right)^8 + C$$

INTEGRAL #9

**READY,
GET SET,...**

2:30

INTEGRAL #9

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

INTEGRAL #9

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

$$= 2 \int \sin u du \quad u = \sqrt{x}, \quad du = \frac{1}{2\sqrt{x}} dx$$

$$= -2 \cos u + C$$

$$= -2 \cos \sqrt{x} + C$$

INTEGRAL #10

**READY,
GET SET,...**

2:30

INTEGRAL #10

$$\int e^x \sec(e^x) \tan(e^x) dx$$

INTEGRAL #10

$$\int e^x \sec(e^x) \tan(e^x) dx$$

$$= \int \sec u \tan u du \quad u = e^x, \quad du = e^x dx$$

$$= \sec u + C$$

$$= \sec(e^x) + C$$

INTEGRAL #11

**READY,
GET SET,...**

2:30

INTEGRAL #11

$$\int \sin x \sin 2x \, dx$$

INTEGRAL #11

$$\int \sin x \sin 2x \, dx$$

$$= \int \sin x \cdot 2 \sin x \cos x \, dx$$

$$= 2 \int \sin^2 x \cos x \, dx$$

$$= \frac{2 \sin^3 x}{3} + C$$

INTEGRAL #12

**READY,
GET SET,...**

2:30

INTEGRAL #12

$$\int \frac{\sin x}{\sqrt{1 + \cos x}} dx$$

INTEGRAL #12

$$\int \frac{\sin x}{\sqrt{1 + \cos x}} dx$$

$$= - \int \frac{1}{\sqrt{u}} du \quad u = 1 + \cos x, \quad du = -\sin x dx$$

$$= -2\sqrt{u} + C$$

$$= -2\sqrt{1 + \cos x} + C$$

INTEGRAL #13

**READY,
GET SET,...**

2:30

INTEGRAL #13

$$\int \frac{x^2}{\sqrt{x+1}} dx$$

INTEGRAL #13

$$\int \frac{x^2}{\sqrt{x+1}} dx$$

$$= \int \frac{(u-1)^2}{\sqrt{u}} du \quad u = x + 1, \quad du = dx$$

$$= \int \frac{u^2 - 2u + 1}{\sqrt{u}} du = \int \left(u^{3/2} - 2u^{1/2} + u^{-1/2} \right) du$$

$$= \frac{2(x+1)^{5/2}}{5} - \frac{4(x+1)^{3/2}}{3} + 2(x+1)^{1/2} + C$$

**READY,
GET SET,...**

2:30

INTEGRAL #14

$$\int \frac{1}{x\sqrt{\ln x}} dx$$

INTEGRAL #14

$$\int \frac{1}{x\sqrt{\ln x}} dx$$

$$= \int \frac{1}{\sqrt{u}} du = \quad u = \ln x, \quad du = \frac{1}{x} dx$$

$$= 2\sqrt{u} + C$$

$$= 2\sqrt{\ln x} + C$$

INTEGRAL #15

**READY,
GET SET,...**

2:30

INTEGRAL #15

$$\int \frac{x^2 + \sec x}{x^2 \sec x} dx$$

INTEGRAL #15

$$\int \frac{x^2 + \sec x}{x^2 \sec x} dx$$

$$= \int \left(\frac{x^2}{x^2 \sec x} + \frac{\sec x}{x^2 \sec x} \right) dx$$

$$= \int \left(\frac{1}{\sec x} + \frac{1}{x^2} \right) dx$$

$$= \int (\cos x + x^{-2}) dx = \sin x - \frac{1}{x} + C$$

**READY,
GET SET,...**

2:30

INTEGRAL #16

$$\int (x + 1) \cos x \, dx$$

INTEGRAL #16

$$\int (x + 1) \cos x \, dx$$

integrate by parts: $u = x + 1$, $dv = \cos x \, dx$

$$= (x + 1) \sin x - \int \sin x \, dx$$

$$= (x + 1) \sin x + \cos x + C$$

INTEGRAL #17

**READY,
GET SET,...**

2:30

INTEGRAL #17

$$\int \frac{\sec \sqrt{x} \tan \sqrt{x}}{\sqrt{x}} dx$$

INTEGRAL #17

$$\int \frac{\sec \sqrt{x} \tan \sqrt{x}}{\sqrt{x}} dx$$

$$= 2 \int \sec u \tan u dx$$

$$u = \sqrt{x}, \quad du = \frac{1}{2\sqrt{x}} dx$$

$$= 2 \sec u + C$$

$$= 2 \sec \sqrt{x} + C$$

**READY,
GET SET,...**

2:30

INTEGRAL #18

$$\int \frac{\ln x}{x^2} dx$$

INTEGRAL #18

$$\int \frac{\ln x}{x^2} dx$$

integrate by parts: $u = \ln x$, $dv = \frac{1}{x^2} dx$

$$= -\frac{\ln x}{x} + \int \frac{1}{x^2} dx$$

$$= -\frac{\ln x}{x} - \frac{1}{x} + C$$

INTEGRAL #19

**READY,
GET SET,...**

2:30

INTEGRAL #19

$$\int \left(\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \right) dx$$

INTEGRAL #19

$$\int \left(\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \right) dx$$

$$= \int (\csc^2 x + \sec^2 x) dx$$

$$= -\cot x + \tan x + C$$

INTEGRAL #20

**READY,
GET SET,...**

2:30

INTEGRAL #20

$$\int \frac{e^x}{e^{2x} + 2e^x + 1} dx$$

INTEGRAL #20

$$\int \frac{e^x}{e^{2x} + 2e^x + 1} dx$$

$$= \int \frac{e^x}{(e^x + 1)^2} dx$$

$$= \int \frac{1}{u^2} du \quad u = e^x + 1, \quad du = e^x dx$$

$$= -\frac{1}{u} + C = \boxed{-\frac{1}{e^x + 1} + C}$$

INTEGRAL #21

**READY,
GET SET,...**

2:30

INTEGRAL #21

$$\int \frac{1}{x \sin^2(\ln x)} dx$$

INTEGRAL #21

$$\int \frac{1}{x \sin^2(\ln x)} dx$$

$$= \int \frac{1}{\sin^2 u} du \quad u = \ln x, \quad du = \frac{1}{x} dx$$

$$= \int \csc^2 u du$$

$$= -\cot u + C$$

$$= -\cot \ln x + C$$

**READY,
GET SET,...**

2:30

INTEGRAL #22

$$\int \sec \sqrt{x} \tan \sqrt{x} dx$$

INTEGRAL #22

$$\int \sec \sqrt{x} \tan \sqrt{x} dx$$

$$= \int 2u \sec u \tan u dx \quad u = \sqrt{x}, \quad u^2 = x, \quad 2u du = dx$$

$$= 2u \sec u - 2 \int \sec u du \quad \text{by parts}$$

$$= 2u \sec u - 2 \ln|\sec u + \tan u| + C$$

$$= 2\sqrt{x} \sec \sqrt{x} - 2 \ln|\sec \sqrt{x} + \tan \sqrt{x}| + C$$

**READY,
GET SET,...**

2:30

INTEGRAL #23

$$\int \frac{\tan^2 \sqrt{x}}{\sqrt{x}} dx$$

INTEGRAL #23

$$\int \frac{\tan^2 \sqrt{x}}{\sqrt{x}} dx$$

$$= 2 \int \tan^2 u \, du$$

$$= 2 \int (\sec^2 u - 1) \, du$$

$$= 2(\tan u - u) + C$$

$$= 2(\tan \sqrt{x} - \sqrt{x}) + C$$

**READY,
GET SET,...**

2:30

INTEGRAL #24

$$\int (1 - \sqrt{x})^{99} dx$$

INTEGRAL #24

$$\int (1 - \sqrt{x})^{99} dx$$

$$u = 1 - \sqrt{x}, \quad x = (u - 1)^2, \quad dx = (2u - 2) du$$

$$= \int u^{99} (2u - 2) du = \frac{2u^{101}}{101} - \frac{u^{100}}{50} + C$$

$$= \frac{2(1 - \sqrt{x})^{101}}{101} - \frac{(1 - \sqrt{x})^{100}}{50} + C$$

INTEGRAL #25

**READY,
GET SET,...**

2:30

INTEGRAL #25

$$\int \sqrt[3]{x^5 + x^3} dx$$

INTEGRAL #25

$$\int \sqrt[3]{x^5 + x^3} dx$$

$$= \int x \sqrt[3]{x^2 + 1} dx$$

$$= \frac{3(x^2 + 1)^{4/3}}{8} + C$$